Multi-frequency Gravitational Wave Astrophysics

Hyung Mok Lee
Korea Astronomy and Space Science Institute (KASI)
and
Chunglee Kim
Ewha Womans University

on behalf of the Korean Gravitational Wave Group (KGWG)

November 6th, 2018
The 8th KIAS Workshop on Cosmology and Structure Formation

Secular Evolution of Binaries in Quadrupole Approximation (Peters 1964)

- The GW frequency is twice of the orbital frequency:

$$f = \frac{2}{P_{orb}}$$

- The orbit averaged rates of changes of semi-major-axis, and eccentricities are

$$\left\langle \frac{da}{dt} \right\rangle = -\frac{64}{5} \frac{G^3 m_1 m_2 (m_1 + m_2)}{c^5 a^3 (1 - e^2)^{7/2}} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right)$$

$$\left\langle \frac{de}{dt} \right\rangle = -\frac{304}{15} e \frac{G^3 m_1 m_2 (m_1 + m_2)}{c^5 a^4 (1 - e^2)^{5/2}} \left(1 + \frac{121}{304} e^2 \right)$$

GW frequency and the mass

- Duration of the frequency at *f* (ignoring the eccentricity)

$$T = \frac{f}{\dot{f}} = \frac{5}{96}\pi^{-8/3}\frac{c^5}{\eta(GM)^{5/3}}f^{-8/3}$$

where η is symmetric mass ratio:

$$\eta \equiv \frac{m_1 m_2}{(m_1+m_2)^2}$$

- We may assume that the merger take place at innermost stable circular orbit (ISCO), then

$$f_{ISCO} \approx \frac{1}{\pi}\left(\frac{1}{6}\right)^{3/2}\frac{c^3}{GM}$$

- The highest frequency for a given binary system is inversely proportional to the mass of the system.

Evolution of GW amplitude during inspiral

- GW amplitude

$$h(t) \propto M^{5/3} D^{-1} f(t)^{2/3} \propto M^{5/4} D^{-1} (t_{coal} - t)^{-1/4}$$

- Fourier transform

$$\tilde{h}(f) \propto M^{5/6} D^{-1} f^{-7/6}$$

- Characteristic Amplitude and power spectral density:

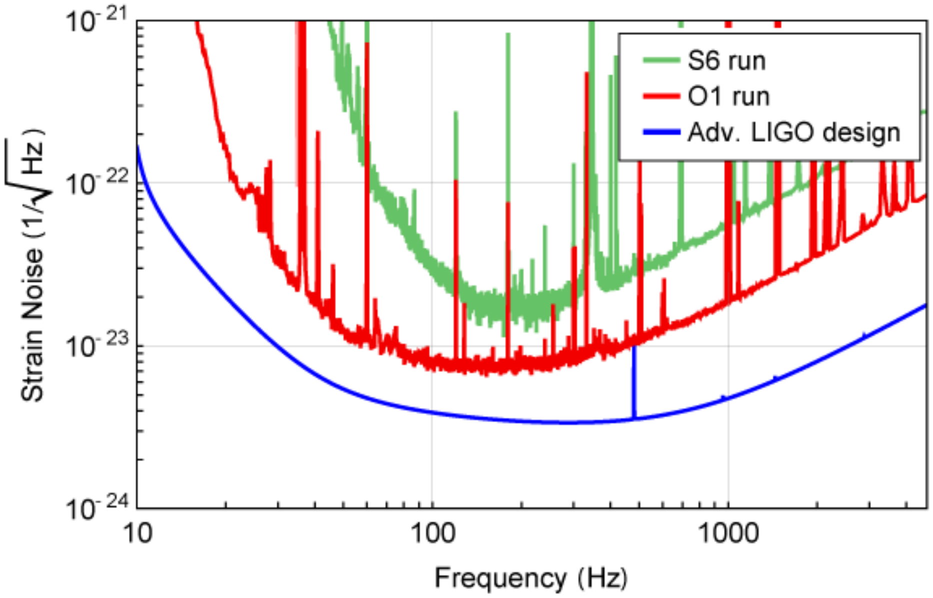
$$h_c(f) = 2f\tilde{h}(f); \quad \sqrt{S_h(f)} = h_c(f) f^{-1/2} = 2f^{1/2}\tilde{h}(f)$$

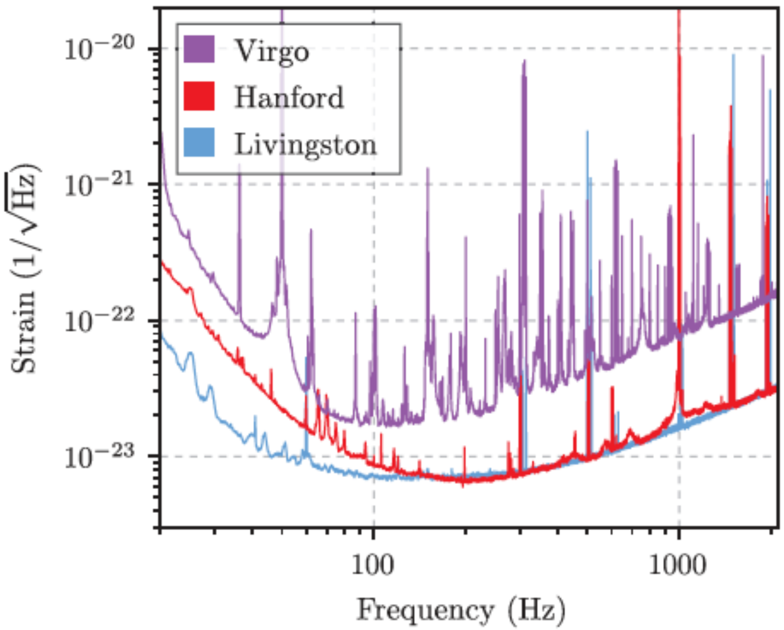
Sources at different frequencies

- High-frequency
    - LIGO/Virgo
    - 30 - 1000 Hz
    - Stellar mass black holes, neutron stars
- Low Frequency
    - LISA: 0.01 mHz ~ 0.1 Hz
        - White dwarf binaries
        - Massive black holes (10^6 M_{sun})
    - Pulsar timing array: nHz
        - Lower frequencies than LISA
        - Supermassive black holes (SMBH, 10^{8-9} M_{sun})
- Mid-Frequency
    - 0.01 - 1 Hz
    - Intermediate mass black holes (IMBH, 10^{3-4} M_{sun})
    - New concepts are being discussed

LIGO Sensitivity during the first and second observing runs [O1/O2]

https://www.advancedligo.mit.edu





2017.08.14

- We expect higher sensitivity in O3 that will start early next year

BBH Detected by the LIGO so far…

	GW150914	LVT1501012	GW151226	GW170104	GW170608	GW170814
m1	36	23	14.2	31.2	12	30.5
m2	29	13	7.5	19.4	7	25.3
m_{final}	62	35	20.8	48.7	18	53.2
S_{final}	0.67	0.66	0.74	0.64	0.69	0.7
mass ratio, q	1.2	1.8	1.9	1.6	1.7	1.2

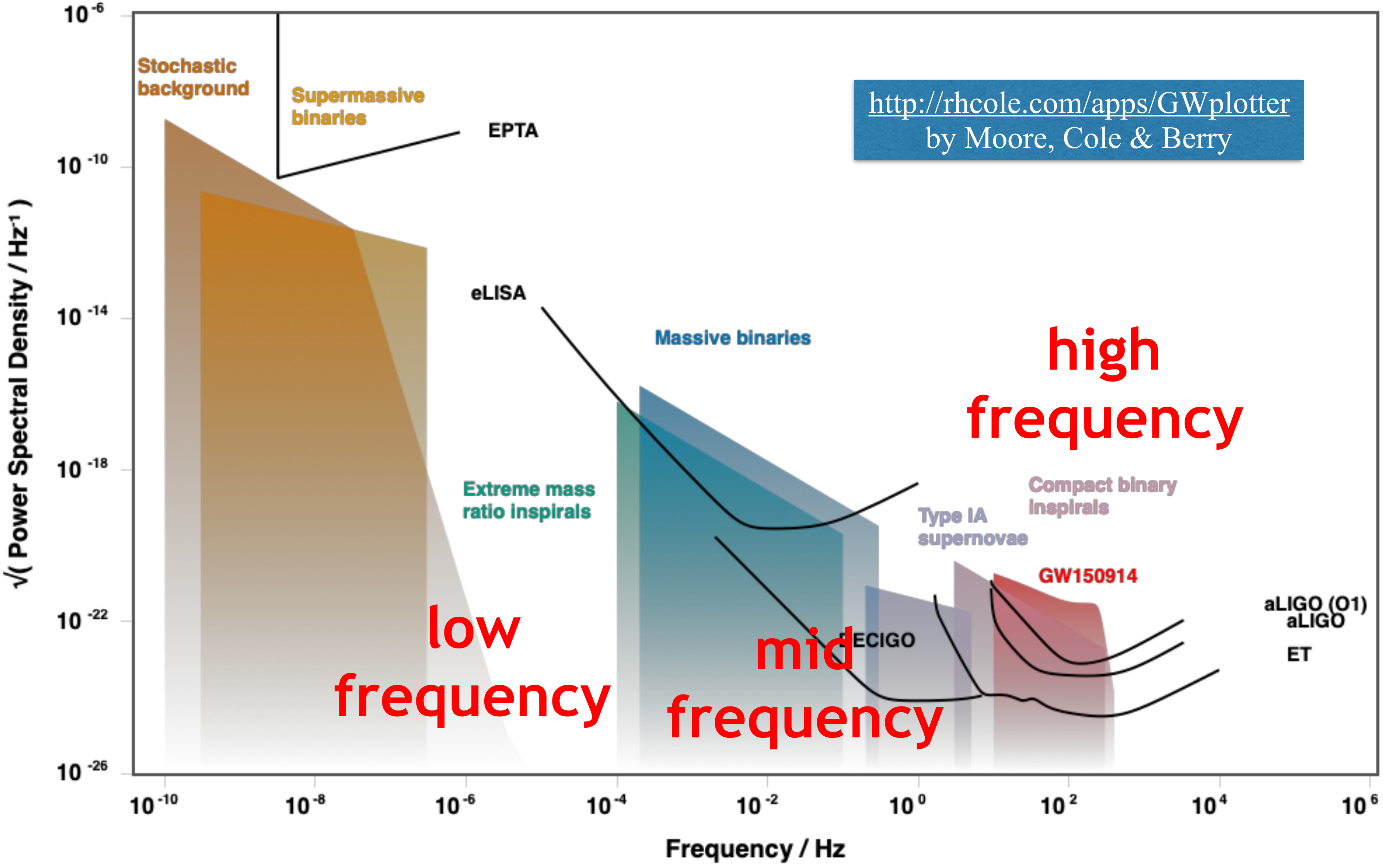
- Current ground-based detectors are sensitive to stellar mass black hole (~10 M_{sun}) binaries.
- Most of the detected sources are nearly equal mass non-spinning binaries in circular orbit.
- Inspiral time scales in LIGO band are < 1 sec.

Proposed Future Detectors

(not a complete list!)

- Ground based
  - Einstein Telescope (ET)
  - Cosmic Explorer (CE)
  - Superconducting Omni-directional Gravittional Radiation Observatory (SOGRO)
  - Mid-band Atomic Gravitational-Wave Interferometric Sensor (MAGIS)
- Space based
  - eLISA (Europe/US)
  - Tianqin and Taiji (China)
  - DECIGO (Japan)

Gravitational Waves in Wide Spectral Range

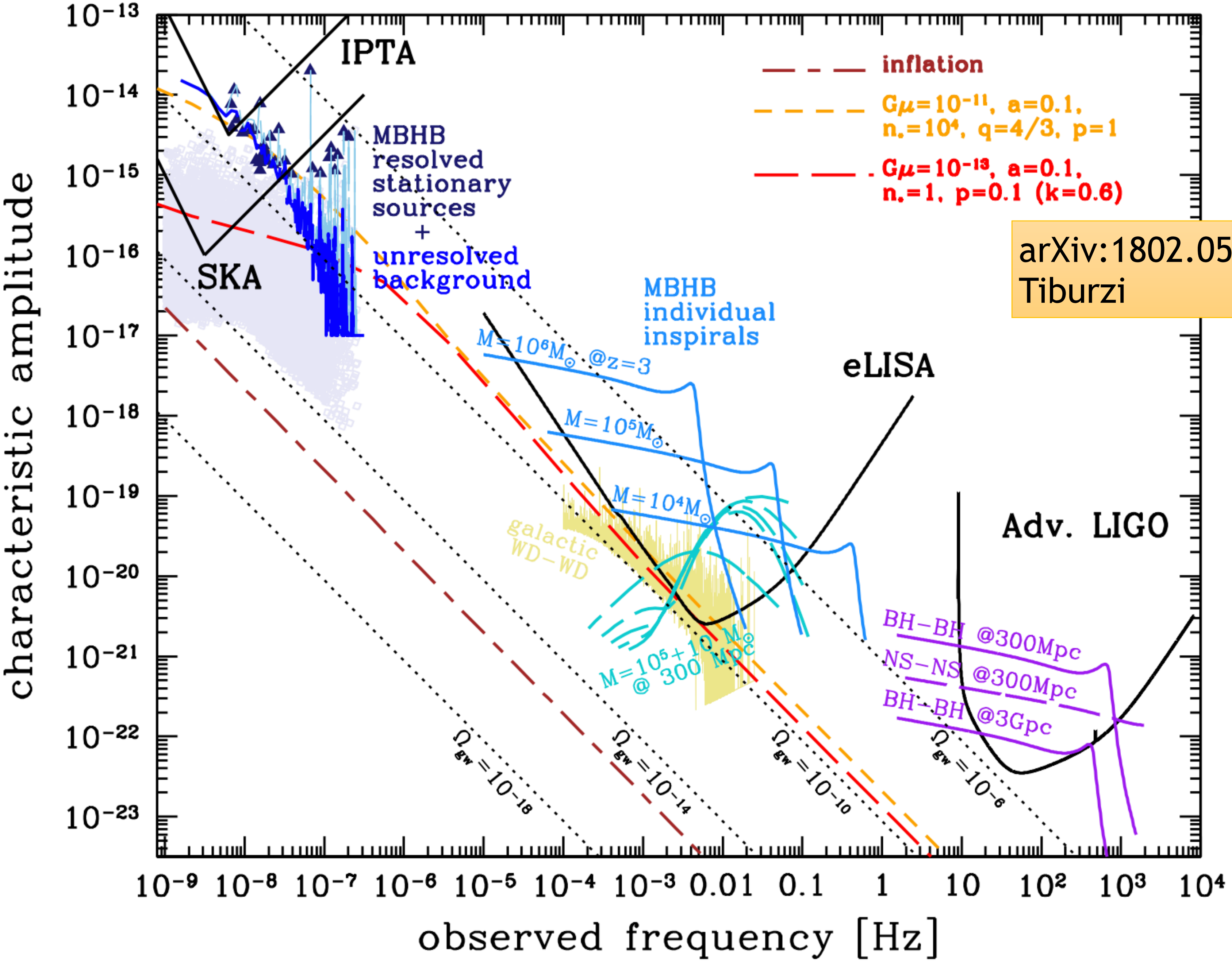


http://rhcole.com/apps/GWplotter
by Moore, Cole & Berry

low frequency

mid frequency

high frequency

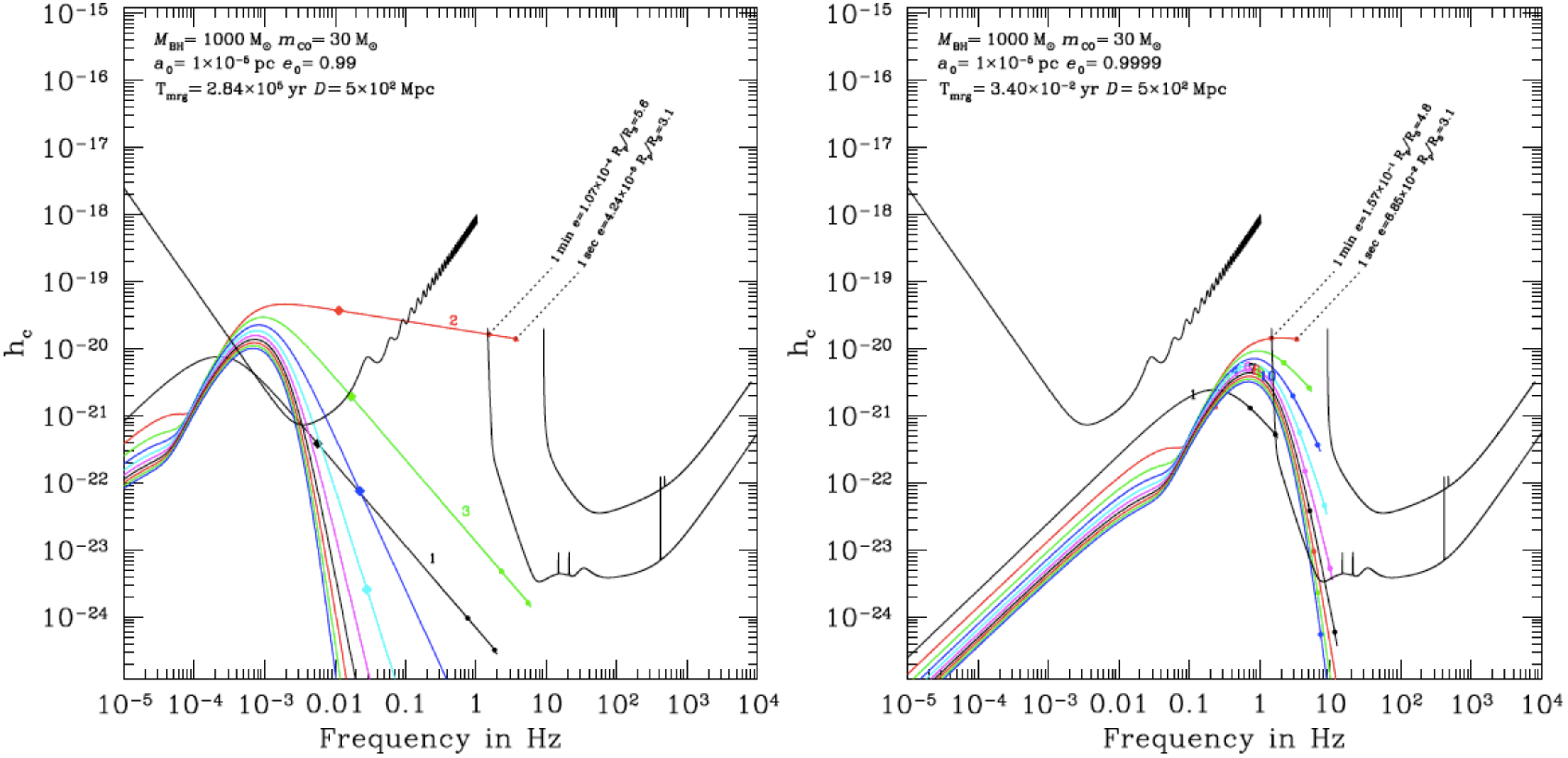


arXiv:1802.05076
Tiburzi

Sources involving massive black holes

- Extreme mass ratio inspiral (EMRI)
  - $q=m_1/m_2$
  - $m_1 \sim 10^6\ M_{sun}$ (massive black hole), $m_2 \sim 1\text{-}10\ M_{sun}$ (NS, stellar mass BH) —> $q > 10^5$
  - Initially very eccentric, many cycles before being swallowed by the MBH
  - mHz GW, galactic nuclei
- Intermediate mass ratio inspiral (IMRI)
  - $m_1 \sim 10^{2\text{-}4}\ M_{sun}$ (Intermediate mass BH)
  - $q \sim 10^{2\text{-}4}$
  - mili to Deci Hz GW, Star Clusters
- Nearly circular binaries

IMRI Waveforms: very sensitive to initial eccentricity



Amaro-Seoane 2018

Mid-frequency ground-based detector

Gravity Gradiometer as a GW Detector

- Geodesic deviation equation: $\dfrac{d^2x^i}{dt^2} = -R^i_{0j0}x^j$
- In weak field limit

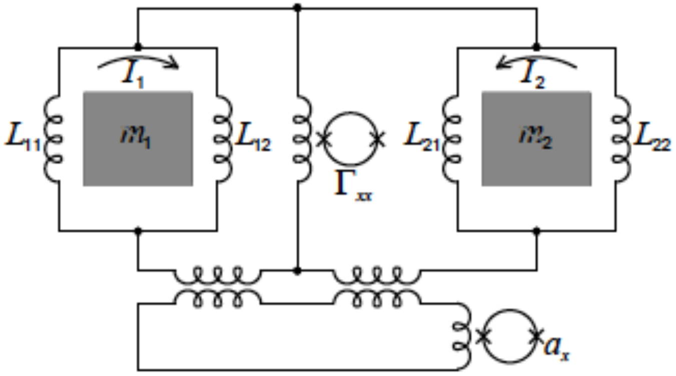
$$R_{i0j0} \approx \frac{\partial^2\phi}{\partial x^i \partial x^j}$$

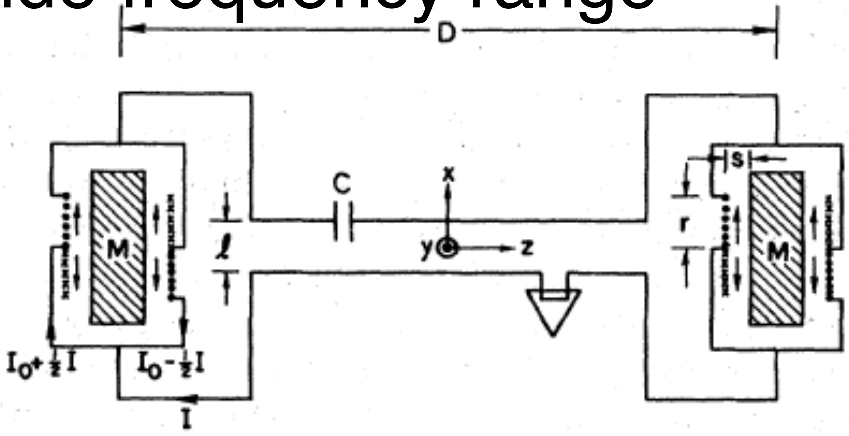
- Strain Amplitude

$$R_{i0j0} = -\frac{1}{2}\frac{\partial^2 h_{ij}}{\partial t^2} \approx \frac{1}{2}\omega^2 h_{ij}$$

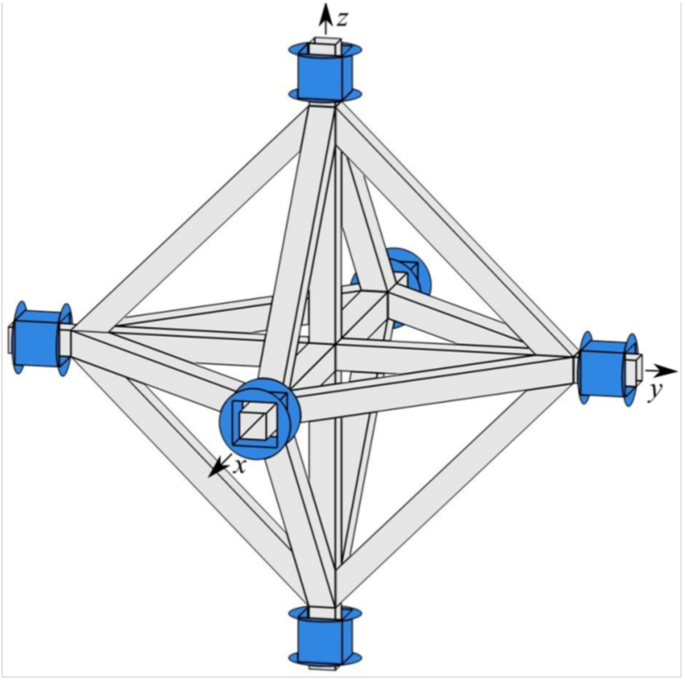
Tunable Free Mass GW Detector (Wagoner et al. 1979)

- The relative motion of two masses induces driving emf of resonant L-C circuit
- The relative momentum is determined by the current in the circuits
- Can be tuned over a wide frequency range





Superconducting tensor GW Detector (Paik et al. 2016, CQG, 33, 075003)



- Superconducting Omni-directional Gravitational Radiation Observatory (SOGRO)

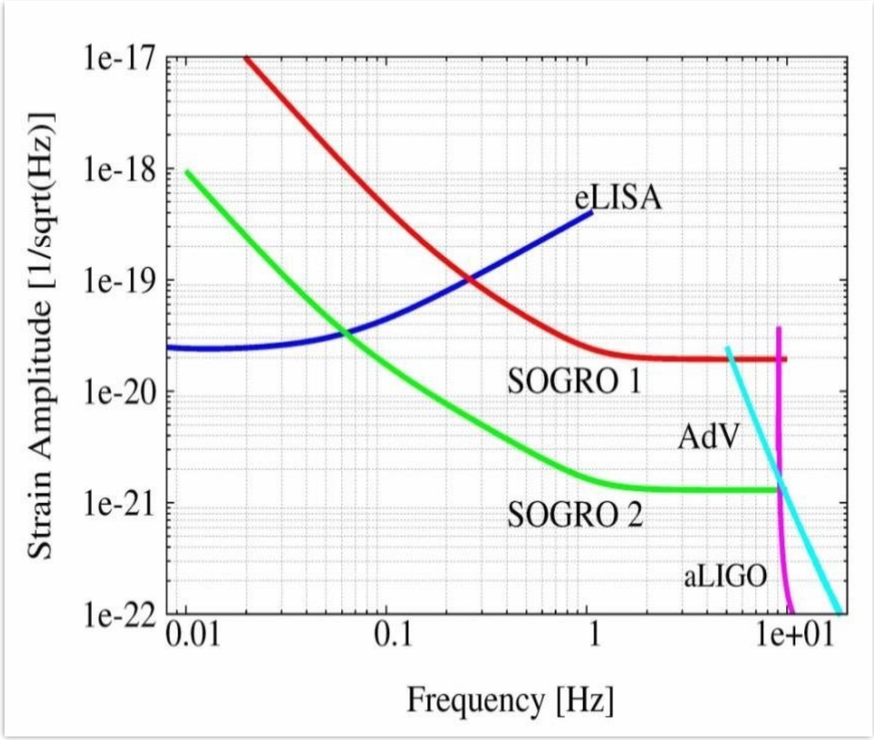
$$h_{ii}(t) = \frac{1}{L}[x_{+ii}(t) - x_{-ii}(t)]$$

$$h_{ij}(t) = \frac{1}{L}\{[x_{+ij}(t) - x_{-ij}(t)] - [x_{-ji}(t) - x_{+ji}(t)]\}$$

- By detecting all six components of Riemann tensor, the source direction and the polarization can be determined

Advantages of SOGRO

- SOGRO would fill in the missing signal band between eLISA and aLIGO/Virgo/KAGRA, 0.1 – 10 Hz.
- SOGRO is a tensor detector with all-sky coverage and with the ability to locate the source and determine wave polarization.
- SOGRO, a full-tensor detector, has an advantage in rejecting Newtonian Noise



Paik et al. 2016, 30m and 100m baseline

Most plausible sources in the mid-frequency GWs

- Intermediate-Mass Black Holes (IMBHs)
- Intermediate-Mass Ratio Inspirals (IMRIs)
- Nearby stellar-mass BBHs ("nearby" for detectors on Earth)
  (total mass < a few hundreds of M_{sun})

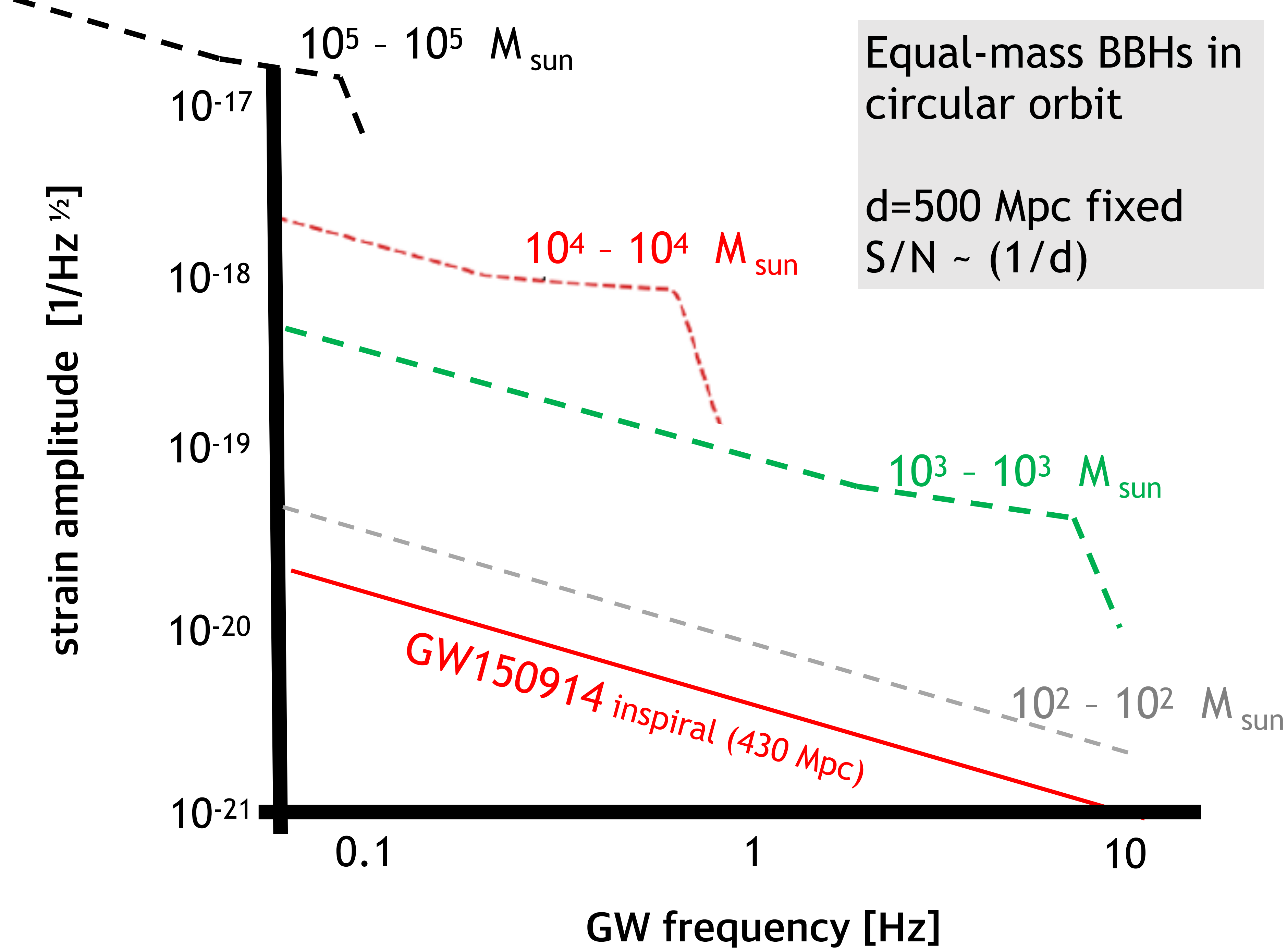
stellar-mass BHs : $\mathcal{O}(1 - 100)\ M_{sun}$

Intermediate-mass BHs : $\mathcal{O}(1000 - 10^5)\ M_{sun}$

- **IMBH astrophysics : questions to be answered**

  - formation scenario of IMBHs
  - evolution of binaries consisting of IMBHs
    (how a IMBH-IMBH binary or an IMRI could be formed ? in what environment?)

- **As GW sources we consider**
  IMBH & IMBH (inspiral-merger-ringdown)
  or IMBH & stellar-mass BH (mainly inspirals)



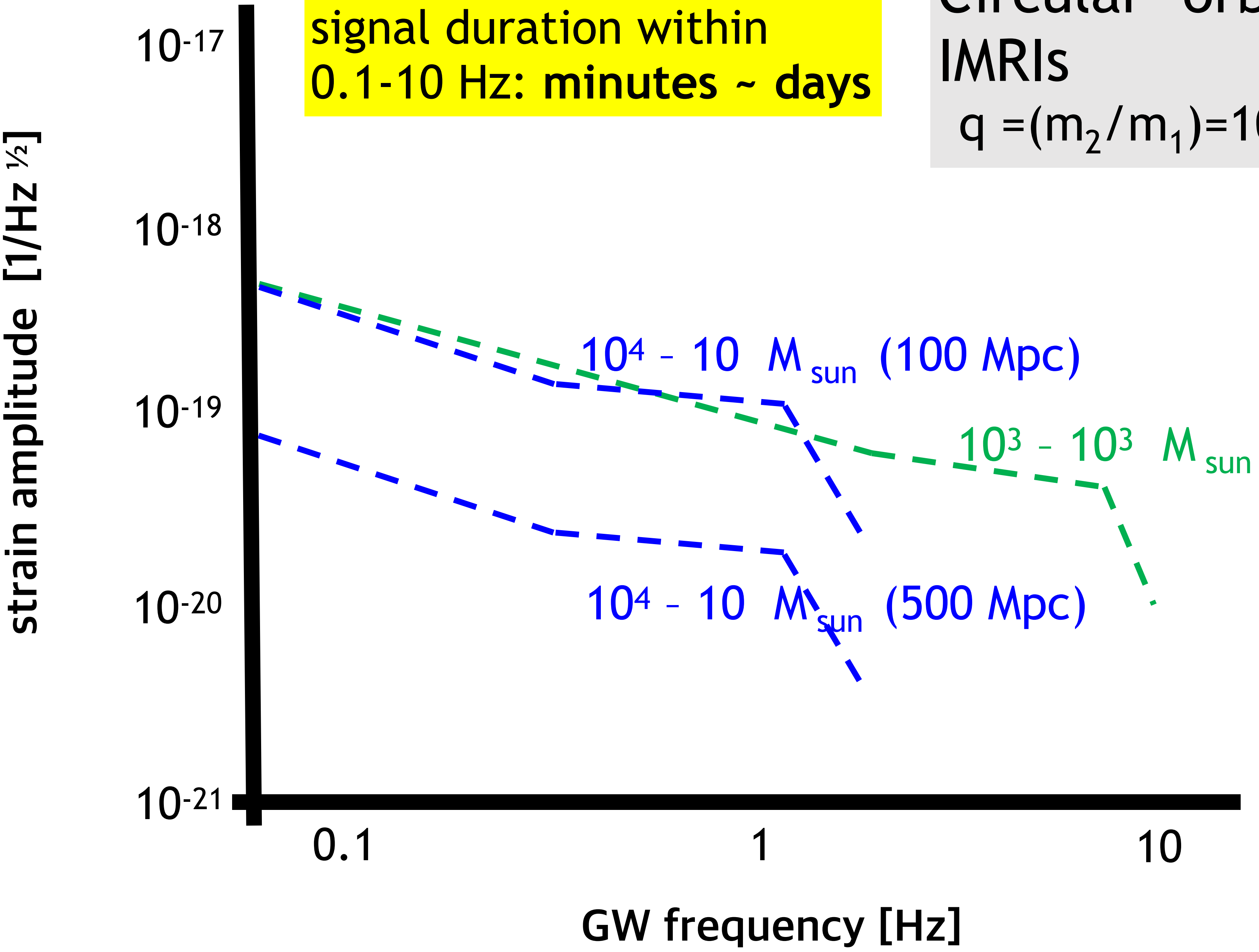
Equal-mass BBHs in circular orbit

d=500 Mpc fixed
S/N ~ (1/d)

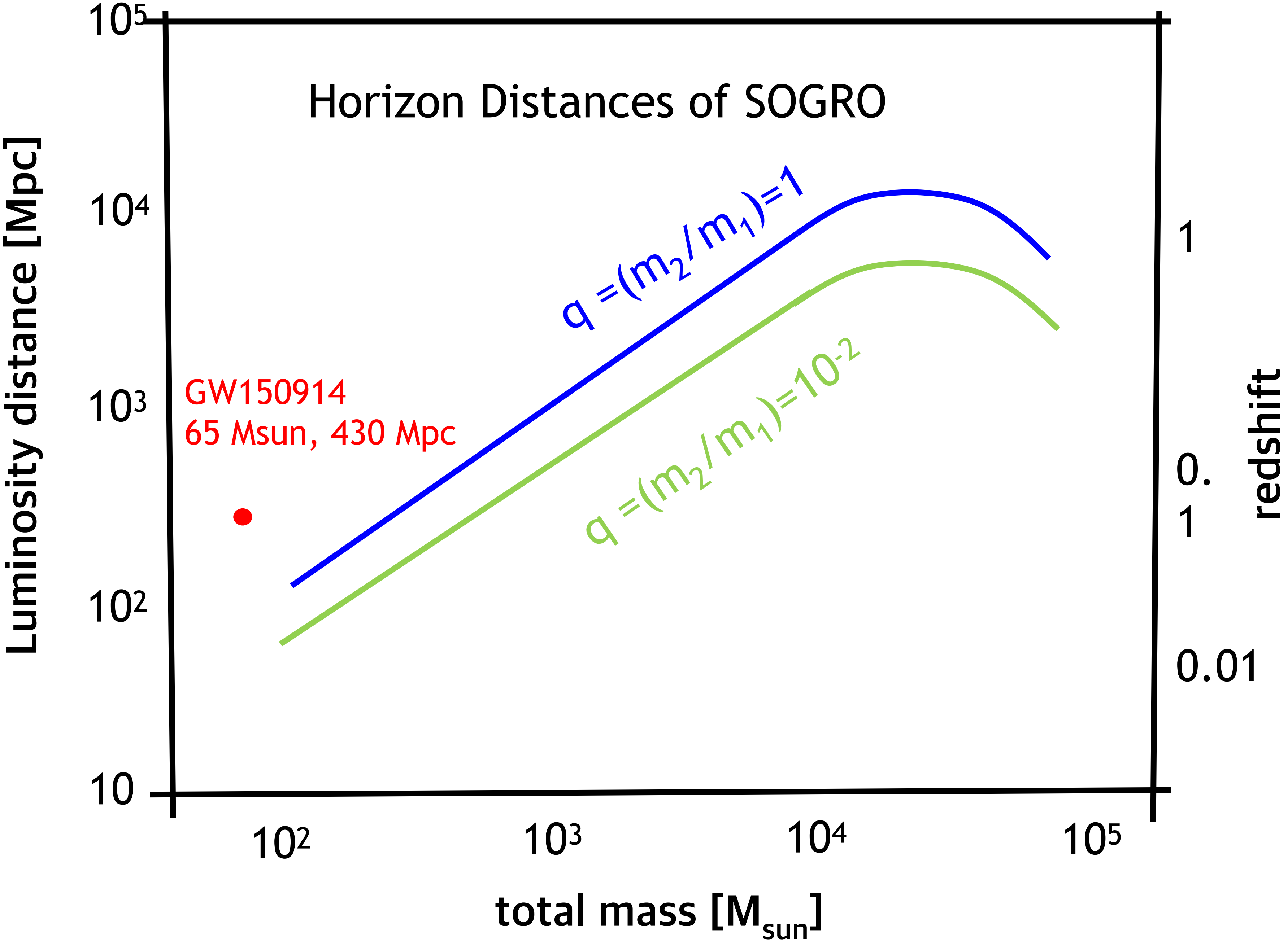
Circular orbit IMRIs

$q = (m_2/m_1) = 10^{-3}$

signal duration within 0.1-10 Hz: **minutes ~ days**



Horizon Distances of SOGRO



Benefit of low to mid-frequency GW astrophysics

- Exploration of the space-time structure of around the (spinning) massive black holes
- Population of compact stars in the central parts of the galaxies
- Horizon distances are much larger than those of high-frequency sources
- Origin of the intermediate mass black holes
- Growth of the massive black holes
- Early warning to the higher frequency detectors

Summary

- Future detectors will cover wider range of gravitational wave frequencies, especially lower than current detectors
- Inspiral of binaries of massive black holes is more complex than compact binary coalescence observed by LIGO.
- Lower frequency detectors can probe new population at large distances
- There are many theoretical challenges, such as accurate waveform modeling, possible perturbation by other stars during inspiral, etc.