KOREA INSTITUTE FOR ADVANCED STUDY

Using the Topology of Large Scale Structure for Cosmological Parameter Estimation

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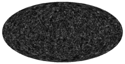

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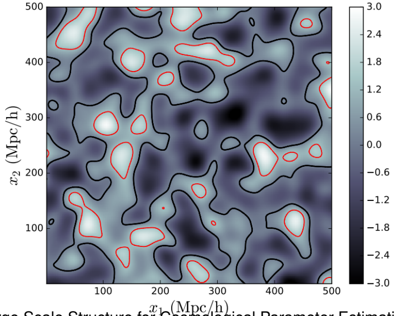
Genus - Definition

- What is the genus of a two-dimensional field?
- A Topological quantity, i.e. it is independent of morphology.
- We define an excursion set of a field using an iso-field boundary (perimeter in two-dimensions).
- Genus definition :

$$G_{2\mathrm{D}} = \text{number of connected regions} - \text{number of holes}$$

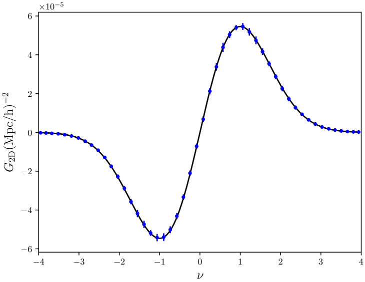


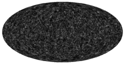
Theoretical Prediction - Genus

- The genus is a function of the density threshold that defines the boundary - ν
- The amplitude contains information on the power spectrum shape (Tomita 1986) :

$$\langle G_{2\mathrm{D}} \rangle = \frac{1}{(2\pi)^{3/2}} \frac{\sigma_1^2}{2\sigma_0^2} \nu e^{-\nu^2/2} = A_{\mathrm{G}} e^{-\nu^2/2} H_1(\nu)$$

$$\sigma_i^2 = \int dk k^{2i+1} P_{2\mathrm{D}}(k)$$





Theoretical Prediction - Genus, Non-Gaussian field

For a non-Gaussian field (such as the low redshift matter density), the leading order non-linear correction due to gravitational collapse can be calculated (Matsubara 1994,2003)

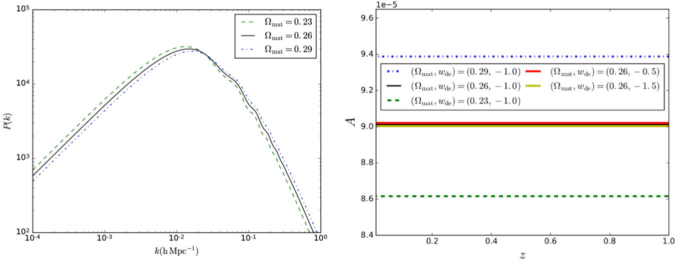
$$G_{2\mathrm{D}}(\nu) = A_{\mathrm{G}} e^{-\nu^2/2} \left[H_1(\nu) + \left(\frac{S^{(0)}}{6} H_3(\nu) + \frac{kS^{(1)}}{3} H_2(\nu) + \frac{k(k-1)S^{(2)}}{6} H_0(\nu) \right) \sigma_0 \right]$$

$$S^{(0)} = \frac{\langle \delta^3 \rangle}{\sigma_0^4} \qquad S^{(1)} = -\frac{3}{4} \frac{\langle \delta^2 \nabla^2 \delta \rangle}{\sigma_0^2 \sigma_1^2} \qquad S^{(2)} = -3 \frac{\langle \nabla \delta . \nabla \delta \nabla^2 \delta \rangle}{\sigma_1^4}$$

- To leading order, the amplitude of the genus is unaffected by non-linear gravitational collapse.
- The amplitude is a measure of the linear matter power spectrum, and so should be conserved with redshift.

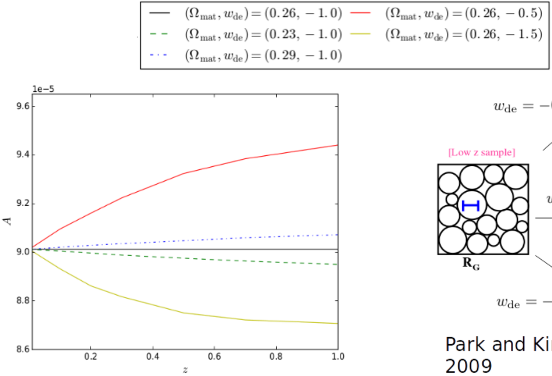
Information Content I – Sensitivity to $\Omega_{\rm cdm}h^2$, $\Omega_{\rm b}h^2$

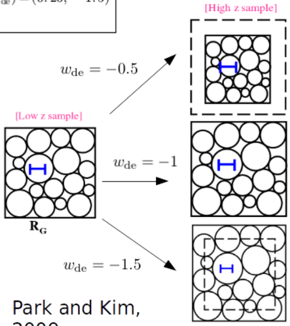
- By measuring the genus of the matter density field, the amplitude is a measure of the shape of the linear matter power spectrum. This is sensitive to $\Omega_{\rm cdm}h^2$, $\Omega_{\rm b}h^2$.
- The genus is insensitive to the amplitude of the power spectrum and hence linear galaxy bias. More small scale power implies a larger genus amplitude.



Information Content II – Conservation with Redshift

- The genus amplitude provides a measure of the linear matter power spectrum – conserved with redshift.
- G_{2D} is a standard population – we find the cosmology that minimizes its evolution with redshift.

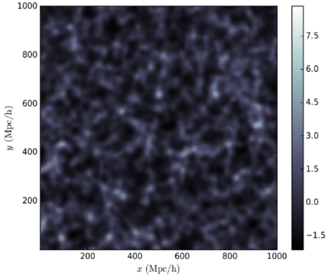


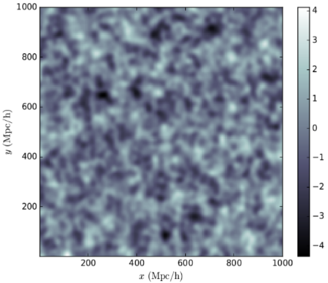


Park and Kim, 2009

Application to Galaxy Data

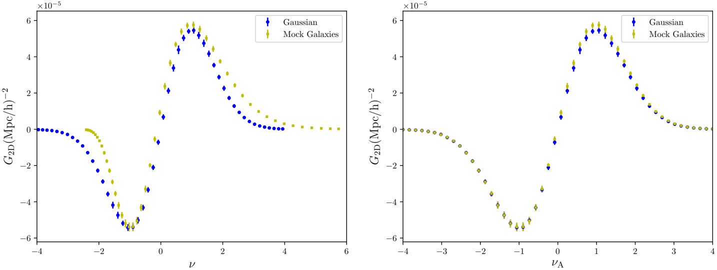
- We use mock galaxy lightcone data from Horizon Run 4.
- HR4 is a $(3150h^{-1}\mathrm{Mpc})^3$ cosmological scale dark matter simulation, 6300^3 particles
- We take all-sky shells of thickness $60h^{-1}\mathrm{Mpc}$ over the redshift range $0.1 < z < 1$.
- A mass cut is applied to fix a constant galaxy number density $\bar{n} = 10^{-3}h^3\mathrm{Mpc}^{-3}$ in each shell.





Gaussian field – Mock Galaxy Comparison

The amplitude of the genus as measured from the mock galaxy catalog is consistent with the Gaussian initial condition.



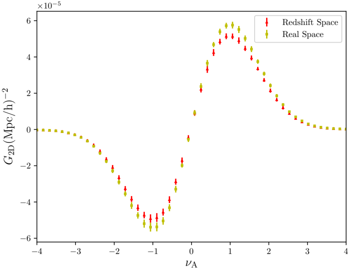
We can re-scale the threshold limit to eliminate the non-Gaussianity of the one-point function.

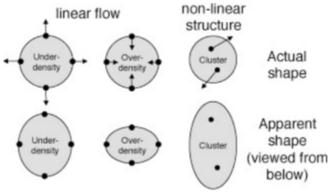
Systematics Removal

- The amplitude of the genus provides a relatively clean measurement of the shape of the linear matter power spectrum.
- However, two significant systematic effects can bias our measurement, and worse introduce a redshift dependence.
- We use snapshot data to study systematics; the most significant are redshift space distortion and shot noise.

Redshift Space Distortion

- For two dimensional slices perpendicular to the line of sight, redshift space distortion creates an effect on structures at the boundaries of the slice.
- Galaxies near over/under-densities are scattered in/out of the slice.
- The effect of (linear) redshift space distortion on the 2D genus has been calculated analytically (Matsubara 1996)





Analytic prediction (Matsubara 1996) –

$$g_{\rm 2D}^{\rm RSD}(\nu, \theta_{\rm s}) = a_{\rm RSD}^{(2D)} g_{\rm 2D}^{\rm real}(\nu)$$

$$a_{\rm RSD}^{\rm (2D)} = \frac{3}{2}\sqrt{\left(1 - \frac{C_1}{C_0}\right)\left[1 - \frac{C_1}{C_0} + \left(\frac{3C_1}{C_0} - 1\right)\cos^2\theta_{\rm s}\right]}$$

$$\frac{C_1}{C_0} = \frac{1}{3}\frac{1 + 6\beta/5 + 3\beta^2/7}{1 + 2\beta/3 + \beta^2/5}$$

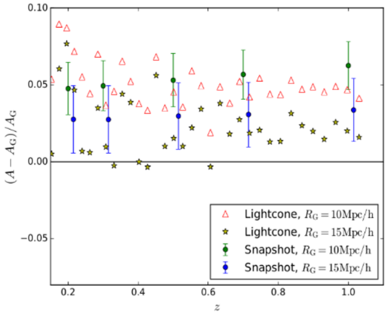
$$\beta = f/b$$

The effect of redshift space distortion re-introduces a bias dependence on the amplitude

We can model the effect of RSD and try to eliminate it, or try to simultaneously constrain cosmological parameters and β

Shot Noise

- Shot noise modifies both the shape and amplitude of the genus curve.
- As we decrease the galaxy number density, structures become increasingly fragmented.
- This effect will artificially increase the number of structures observed.

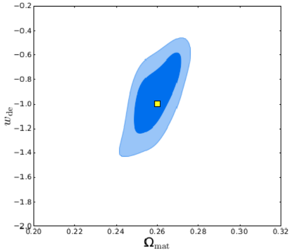


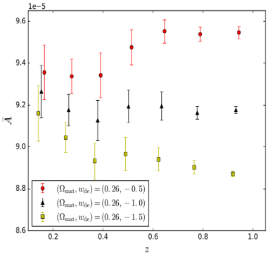
Cosmological Parameter Estimation

- Example - We measure the genus of two dimensional shells of the Horizon Run 4 mock galaxy lightcone (all sky).
- Compare measured genus amplitude to Gaussian expectation value, after correcting for systematics

$$\chi^2_{\rm mag} = \sum_i \frac{\left(A_i(z_i, \Omega_{\rm mat}, w_{\rm de})(1-\Delta_{\rm SN})/a^{(2D)}_{\rm RSD} - A_{\rm G}(\Omega_{\rm mat}, w_{\rm de})\right)^2}{\sigma_i^2 + \sigma^2_{\rm RSD} + \sigma^2_{\rm SN}}$$

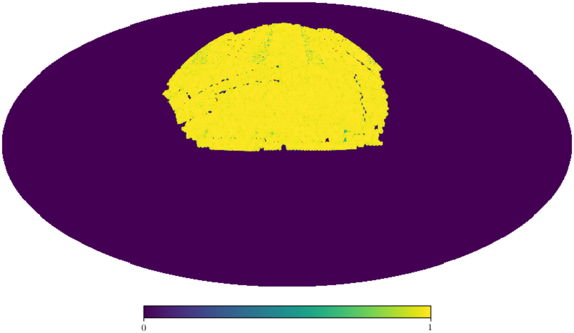
Parameter	Mag
$\Omega_{\rm mat}$	$0.2616^{+0.009}_{-0.009}$
$w_{\rm de}$	$-1.08^{+0.16}_{-0.30}$





Data Systematics - Mask

- When we generate a density field from the SDSS DR12 galaxy data, we must also account for the effect of the mask
- We smooth the field in Fourier space. The mask is sharp in real space - generates spurious oscillations in Fourier Space
- We apodize the mask, smoothing the sharp boundary



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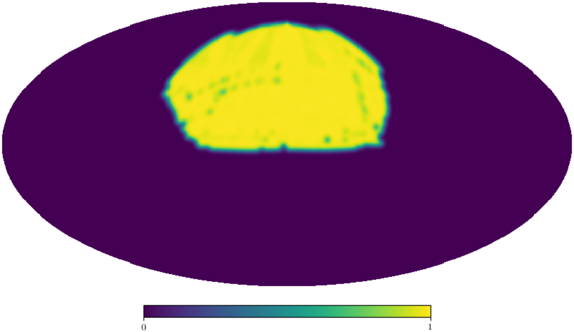
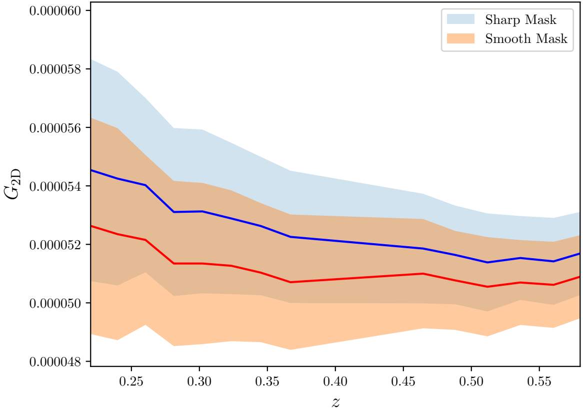


Data Systematics - Mask

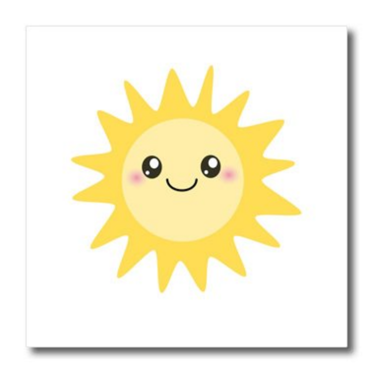
- A sharp mask artificially raises the genus amplitude
- Application of the smoothed mask eliminates this spurious numerical artifact



Summary

- The genus provides a measurement of the shape of the linear matter power spectrum and also potentially the bispectrum.
- These statistic is insensitive to (linear) galaxy bias and non-linear gravitational collapse, down to scales $R_{\rm G} \sim 10h^{-1}{\rm Mpc}$.
- Redshift space distortion can modify the statistics. We can use combinations of two and three-dimensional genus measurements to simultaneously constrain the growth rate and cosmoligical parameters.
- Application to current generation LSS surveys is on-going.

Thank You!!



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